[11:00-11:50] Review of midterm #1

Midterm 1.2(b) Fall 2025

$$e^{-j2\pi f_0 t} = \cos(2\pi f_0 t) + j\sin(2\pi f_0 t)$$

$$y(t) = e^{-j2\pi f_0 t} x(t)$$

$$= (\cos(2\pi f_0 t) + j\sin(2\pi f_0 t))\, x(t)$$

$$= \underline{\cos(2\pi f_0 t)\, x(t)} + j\, \underline{\sin(2\pi f_0 t)\, x(t)}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k\omega_0 t}$$

$$y(t) = e^{-j2\pi f_0 t}\left(\sum_{k=-\infty}^{\infty} a_k e^{j2\pi k\omega_0 t}\right)$$

$$= \sum_{k=-\infty}^{\infty} e^{-j2\pi f_0 t} a_k e^{j2\pi k\omega_0 t}$$

$$y(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 (k-1) t}$$

$$\omega_0 = 2\pi f_0$$

Let $\cancel{m = k+1,\ k = m-1}$
$m = k-1,\ k = m+1$

$$y(t) = \sum_{m=-\infty}^{\infty} a_{m+1} e^{j2\pi f_0 (m) t}$$

$$\longrightarrow b_k = a_{k+1}$$

[11:50-12:00] Discrete time frequency response

An LTI system is uniquely defined by its impulse response $h[n]$

Convolution gives the output $y[n]$ from the input $x[n]$ and the impulse response $h[n]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

Deconvolution gives either $h[n]$ from $\{y[n], x[n]\}$ or gives $x[n]$ from $\{y[n], h[n]\}$

When the input to an LTI system is sinusoidal, the output is also sinusoidal with the same frequency:

$$x[n] = \cos(\hat{\omega}_0 n) \rightarrow y[n] = A\cos(\hat{\omega}_0 n + \theta)$$

Where A is the magnitude response of the system and θ is the phase response. A and θ are functions of the frequency $\hat{\omega}_0$.

Example: Lowpass filter

$$\text{magnitude response} = A = \begin{cases} \text{approximately one} & \text{for low frequencies} \\ \text{approximately zero} & \text{for high frequencies} \end{cases}$$

[12:00-] Sinusoidal response of FIR filters.

Assume the input is $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$ for $-\infty < n < \infty$

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\ &= \sum_{k=0}^{M} h[k]Ae^{j\phi}e^{j\hat{\omega}(n-k)} \\ &= Ae^{j\phi}e^{j\hat{\omega}n} \underbrace{\sum_{k=0}^{M} h[k]e^{-j\hat{\omega}k}}_{\substack{\text{frequency response} \\ \mathcal{H}(\hat{\omega})}} \end{aligned}$$

Matrices
Square matrix A ($n \times n$)

$$\vec{y} = A\vec{x}$$

In certain cases for $\vec{x}$,

$$\vec{y} = A\vec{x} = \underbrace{\lambda}_{\text{scalar}} \vec{x}$$

λ is the eigenvalue for the eigenvetor $\vec{x}$ assuming that A has an eigendecomposition

[12:10] Frequency response of FIR Filters

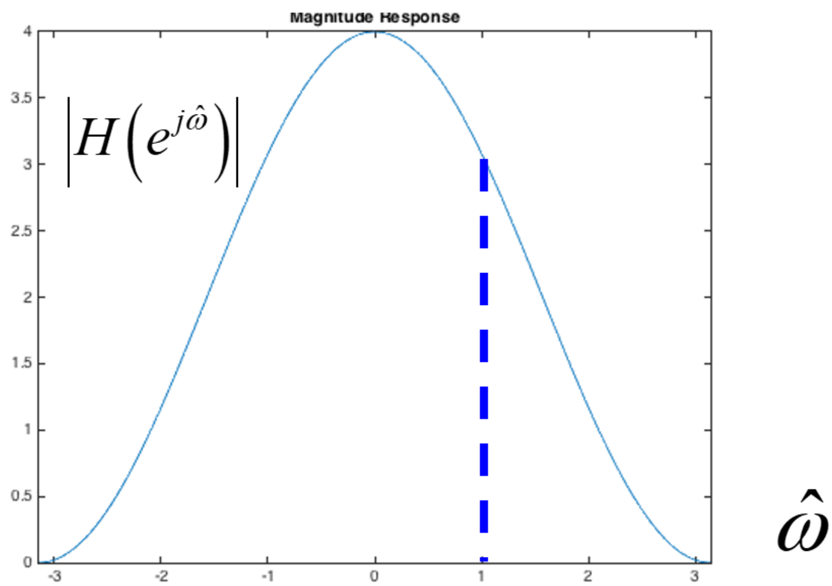
$$\left(e^{j\theta}\right)^n = e^{j\theta n} \qquad \text{(De Moivre's formula)}$$

$$\left(e^{-j\hat{\omega}}\right)^2 = e^{-j\hat{\omega}}e^{-j\hat{\omega}} = e^{-j2\hat{\omega}}$$

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} h[k]e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k]\left(e^{j\hat{\omega}}\right)^{-k} = H\left(e^{j\omega}\right)$$

Example: $h = [0,1,2,1,0,0]$

$$H\left(e^{j\hat{\omega}}\right) = \sum_{k=0}^{M} h[k]\left(e^{j\hat{\omega}}\right)^{-k} = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$



Slide 9-3 10/21/25

$x[n]$ → LTI System $h[n]$ → $y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$

$\cos(\hat{\omega}_0 n)$ → $A\cos(\hat{\omega}_0 n + \theta)$

a function of $\hat{\omega}_0$

A is the magnitude response of LTI system

θ is the phase " " " "

E.g. lowpass filter: A will be approximately one for low frequencies and 0 for high frequencies

Matrices

Square matrix A $n \times n$

$$\vec{y} = A\vec{x}$$

In certain cases for $\vec{x}$,

$$\vec{y} = A\vec{x} = \underbrace{\lambda}_{\text{complex scalar}}\vec{x}$$

λ is the eigenvalue for eigenvector $\vec{x}$ assuming that A has an eigendecomposition